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THE THREE-DIMENSIONAL PROBLEM OF STEADY OSCILLATIONS OF AN ELASTIC HALF-SPACE WITH A SPHERICAL CAVITY*

T.G. RUMYANTSEVA, T.N. SELEZNEVA and M.G. SELEZNEV

The three-dimensional problem of the dynamic theory of elasticity concerning steady harmonic oscillations of an elastic half-space with a spherical cavity is considered. The problem is reduced, with help of the superposition principle, to that of solving a system of six integral equations describing the stress-strain state of the medium. An algorithm for solving the system is given, which can be used in the case when the cavity has a relatively small radius to obtain an approximate solution with any desired degree of accuracy, in the form of an asymptotic expansion. A numerical analysis of the stress-strain state of the elastic medium is given for a wide range of frequencies.

1. Consider the problem of the forced steady harmonic oscillations of an elastic half-space with a deeply placed spherical cavity, in the three-dimensional formulation. The region occupied by the elastic medium is defined by

$$\bar z \geqslant 0,\ \bar r \geqslant a\ (\sqrt{(\bar z + h)^2 + x^2 + y^2} = \bar r)$$

where a is the cavity radius, h is the depth of its centre, $\bar x, \bar y, \bar z$ are rectangular Cartesian coordinates and $\bar r, \alpha, \beta$ are spherical coordinates attached to the cavity centre. Let x, y, z, r denote the dimensionless coordinates referred to the cavity radius a.

The following boundary conditions are specified at the boundary of the cavity in the general case:

$$z = 0,\quad \tau_{xz} = t_1(x, y)\,e^{-i\omega t},\quad \tau_{yz} = t_2(x, y)\,e^{-i\omega t},\quad \sigma_z = t_3(x, y)\,e^{-i\omega t} \tag{1.1}$$
$$r = 1,\quad \sigma_r = \tau_1(\alpha, \beta)\,e^{-i\omega t},\quad \tau_{r\alpha} = \tau_2(\alpha, \beta)\,e^{-i\omega t},\quad \tau_{r\beta} = \tau_3(\alpha, \beta)\,e^{-i\omega t}$$

The motion of the medium is described by the dynamic equations of the theory of elasticity in terms of the displalements, i.e. by the Lamé equations /1/.

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The solution of the boundary value problem formulated here is constructed, as in /2/, in the form of the sum of solutions of two boundary value problems. These are: the problem of steady harmonic oscillations of an elastic half-space under the action of stresses specified at its surface

$$z=0,\quad \tau_{xz}=X_1(x,y)\,e^{-i\omega t},\quad \tau_{yz}=X_2(x,y)\,e^{-i\omega t}$$
$$\sigma_z=X_3(x,y)\,e^{-i\omega t}$$

and the problem of the steady harmonic oscillations of an infinite elastic space with a spherical cavity under a load of distributed forces

$$r=1,\quad \sigma_r=Y_1(\alpha,\beta)\,e^{-i\omega t},\quad \tau_{r\alpha}=Y_2(\alpha,\beta)\,e^{-i\omega t}$$
$$\tau_{r\beta}=Y_3(\alpha,\beta)\,e^{-i\omega t}$$

To determine the unknown functions $X_j(x,y)$, $Y_j(\alpha,\beta)$ $(j=1,2,3)$, we use the boundary conditions of the initial boundary value problem. Adding together the solutions of the two boundary value problems formulated above and satisfying the boundary conditions (1.1), we obtain a system of six integral equations

$$X_j(x,y)=t_j(x,y)-\int_0^{2\pi}\int_0^{\pi}\sum_{k=1}^{3}Y_k(\varphi,\psi)\sum_{n=0}^{\infty}L_{kn}^{(j)}(x,y,\varphi,\psi)\,d\varphi\,d\psi \tag{1.2}$$

$$Y_j(\alpha,\beta)=\tau_j(\alpha,\beta)-\frac{1}{4\pi^2}\sum_{n=1}^{3}\iint_{-\infty}^{\infty}X_n(\zeta,\tau)\times$$
$$\exp\left[i(\xi\zeta+\eta\tau)\right]\int_{\sigma_1}\int_{\sigma_2}K_{jn}(\xi,\eta,\alpha,\beta)\,d\xi\,d\eta\,d\zeta\,d\tau,\quad j=1,2,3$$

Here $\varepsilon=a/h$. The functions $L_{kn}^{(j)}(x,y,\varphi,\psi)$, $K_{jn}(\xi,\eta,\alpha,\beta)$ are very cumbersome, e.g.

$$L_{11}^{(1)}(x,y,\varphi,\psi)=\sum_{n=0}^{\infty}\sum_{k=-n}^{n}\Psi_{11}^{(k)}(x,y)\,\bar S_n^{(k)}(\varphi,\psi)\sin\varphi$$

$$K_{11}(\xi,\eta,\alpha,\beta)=\Big[\Big\langle-\frac{i\xi\lambda_2}{\Delta u^2}\{(\xi^2\cos^2\beta+\eta^2\sin^2\beta+\xi\eta\sin 2\beta)[(\lambda_2^2+$$
$$u^2)\exp(\lambda_2 z_0)-2u^2\exp(\lambda_1 z_0)]+u^2[(\lambda_2^2+u^2)\exp(\lambda_2 z_0)-$$
$$2\lambda_1^2\exp(\lambda_1 z_0)]\}-\frac{i\eta}{\lambda_2 u^2}\left[\xi\eta\cos 2\beta+(\eta^2-\xi^2)\frac{\sin 2\beta}{2}\right]\times$$
$$\exp(\lambda_2 z_0)\Big\rangle(1-\cos 2\alpha)+\Big\{\frac{\xi}{\Delta u^2}[(\lambda_2^2+u^2)^2\exp(\lambda_2 z_0)-$$
$$4\lambda_1\lambda_2 u^2\exp(\lambda_1 z_0)](\xi\cos\beta+\eta\sin\beta)+$$
$$\frac{\eta}{u^2}\exp(\lambda_2 z_0)(\eta\cos\beta-\xi\sin\beta)\Big\}\sin 2\alpha+$$
$$\frac{2i\lambda\xi\lambda_2}{\Delta}(u^2+\lambda_2^2)(\exp(\lambda_2 z_0)-\exp(\lambda_1 z_0))\Big]\exp[-i(\xi x_0+\eta y_0)]$$

$$\Psi_{11n}^{(k)}(x,y)=[a_{22}^{(k)}F_{11n}^{(k)}(x,y)-a_{21}^{(k)}F_{12n}^{(k)}(x,y)]/\Delta_n$$

$$F_{11n}^{(k)}(x,y)=-\frac{1}{\sqrt{r_0}}H_{n+1/2}^{(1)}(\theta_1 r_0)\,S_n^{(k)}(\alpha_0,\beta_0)\sin 2\alpha_0\cos\beta_0$$

$$F_{12n}^{(k)}(x,y)=-\frac{1}{n(n+1)\sqrt{r_0}}H_{n+1/2}^{(1)}(\theta_2 r_0)\left[\frac{\partial S_n^{(k)}}{\partial\alpha}\cos 2\alpha_0\cos\beta_0-\right.$$
$$\left.\cos\alpha_0\sin\beta_0(\partial S_n^{(k)}/\partial\beta)/\sin\alpha_0\right]_{\alpha=\alpha_0,\,\beta=\beta_0}$$

$$r_0=[x^2+y^2+\varepsilon^{-2}]^{1/2},\quad \alpha_0=\operatorname{arctg}\sqrt{x^2+y^2},\quad \beta_0=\operatorname{arctg}(y/x)$$
$$\Delta_n=a_{11}^{(n)}a_{22}^{(n)}-a_{12}^{(n)}a_{21}^{(n)},\quad \Delta=(\lambda_2^2+u^2)^2-4\lambda_1\lambda_2 u^2$$
$$u^2=\xi^2+\eta^2,\quad \lambda_j^2=u^2-\theta_j^2,\quad z_0=\cos\alpha-\varepsilon^{-1},\quad x_0=\sin\alpha\cos\beta$$
$$y_0=\sin\alpha\sin\beta$$
$$a_{11}^{(n)}=[-\theta_2^2/(2\theta_1^2)+n(n-1)/\theta_1^2]\,H_{n+1/2}^{(1)}(\theta_1)+2H_{n+3/2}^{(1)}(\theta_1)/\theta_1$$
$$a_{12}^{(n)}=[(n-1)\,H_{n+1/2}^{(1)}(\theta_2)-\theta_2 H_{n+3/2}^{(1)}(\theta_2)]/\theta_2^2$$
$$a_{21}^{(n)}=n(n+1)[(n-1)\,H_{n+1/2}^{(1)}(\theta_1)-\theta_1 H_{n+3/2}^{(1)}(\theta_1)]/\theta_1^2$$
$$a_{22}^{(n)}=[(n^2-1-\theta_2^2/2)\,H_{n+1/2}^{(1)}(\theta_2)+\theta_2 H_{n+3/2}^{(1)}(\theta_2)]/\theta_2^2$$
$$S_n^{(k)}(\alpha,\beta)=[(2n+1)(n-k)!/\{4\pi(n+k)!\}]^{1/2}P_n(\cos\alpha)\exp(ik\beta)$$
$$\theta_1^2=\rho\omega^2a^2/(\lambda+2\mu),\quad \theta_2^2=\rho\omega^2a^2/\mu$$

where $\bar S_n^{(k)}(\alpha,\beta)$ are functions conjugate to $S_n^{(k)}(\alpha,\beta)$.

The contours of integration σ_1,σ_2 are chosen taking into account the principle of limit absorption /3/, and the solution of the system is constructed in the class of summable functions.

Analysing the elements of the system of integral Eqs.(1.2) we find that when $h > a$, the operator of the system is completely continuous in the space of summable functions, and we can, in general, reduce the system, as in /2/, to an infinite quasiregular system of linear algebraic equations whose solutions can be obtained using a computer, e.g. by a reduction method.

When ε are small, the operator of the system will be compressive, and this enables us to use the method of consecutive approximations with asymptotic computation of the integrals to obtain its solution. The solution is obtained in the form of an asymptotic expansion in powers of the small parameter ε

$$X_j = X_{j0} + \varepsilon X_{j1} + \ldots, \quad Y_j = Y_{j0} + \varepsilon Y_{j1} + \ldots$$

Let us consider the case when $\varepsilon \ll 1$. Analysis of the properties of the elements of system (1.2) determines the choice of the zero-order approximation

$$X_{j0}(x, y) = t_j(x, y), \quad Y_{j0}(\alpha, \beta) = \tau_j(\alpha, \beta), \quad j = 1, 2, 3$$

In order to construct the higher approximations, we will write the functions t_j, τ_j in a more specific form such as

$$t_1(x, y) = t_2(x, y) = 0, \quad t_3(x, y) = P(x, y) = \begin{cases} p = \text{const}, & x, y \in \Omega \\ 0, & x, y \overline{\in} \Omega \end{cases}$$

$$\tau_j(\alpha, \beta) = 0, \; j = 1, 2, 3, \; \Omega : x \in [b_1, b_2] \cup y \in [c_1, c_2]$$

Calculating the first-order approximation with the accuracy of up to $O(\varepsilon^2)$, we obtain

$$Y_j(\alpha, \beta) = \frac{2ip\varepsilon}{\pi} \sum_{k=1}^{2} \frac{1}{MN\theta_k} \sin(BM_*\theta_k) \sin(CN_*\theta_k) \times$$
$$f_{jk}(\alpha, \beta, M_*\theta_k, N_*\theta_k) \exp\left[i\left(\frac{\theta_k}{\varepsilon}\sqrt{1 + M^2 + N^2} - \theta_k \cos\alpha\right)\right] +$$
$$O(\varepsilon^2)$$
$$X_k(x, y) = O(\varepsilon^2), \quad k = 1, 2, \quad j = 1, 2, 3, \quad X_3 = P(x, y) + O(\varepsilon^2)$$
$$M_* = \frac{M}{\sqrt{1 + M^2 + N^2}}, \quad N_* = \frac{N}{\sqrt{1 + M^2 + N^2}}, \quad b_2 \neq b_1, \quad c_2 \neq c_1$$
$$M = \varepsilon(b_1 + b_2)/2, \; N = \varepsilon(c_1 + c_2)/2, \; B = (b_2 - b_1)/2$$
$$C = (c_2 - c_1)/2$$

The expressions for the terms $O(\varepsilon^2)$ (the second approximation) are very cumbersome, and are therefore omitted. The following notation is used in the above relations:

$$f_{11}(\alpha, \beta, \xi, \eta) = -\frac{\gamma_2^2}{\Delta}\left\{\frac{1}{2} R_3(1 - \cos 2\alpha) + 2i\lambda_1 R_2 \sin 2\alpha -\right.$$
$$[\lambda_1\theta_2/\theta_1]^2 + \lambda u^2/\mu, \quad \gamma_2^2 = u^2 + \lambda_2^2$$
$$f_{12}(\alpha, \beta, \xi, \eta) = \frac{2\lambda_1\lambda_2}{\Delta}\left\{\frac{1}{2} R_1(1 - \cos 2\alpha) + \frac{i}{\lambda_2} R_2\gamma_2^2 \sin 2\alpha - 2u^2\right\}$$
$$f_{21}(\alpha, \beta, \xi, \eta) = -\frac{\gamma_2^2}{\Delta}\left\{\frac{1}{2} R_3 \sin 2\alpha + 2i\lambda_1 R_2 \cos 2\alpha\right\}$$
$$f_{22}(\alpha, \beta, \xi, \eta) = \frac{2\lambda_1\lambda_2}{\Delta}\left\{\frac{1}{2} R_1 \sin 2\alpha + \frac{i\gamma_2^2}{\lambda_2} R_2 \cos 2\alpha\right\}$$
$$f_{31}(\alpha, \beta, \xi, \eta) = -\frac{\gamma_2^2}{\Delta}(R_4 + 2i\lambda_1 R_5)$$
$$f_{32}(\alpha, \beta, \xi, \eta) = \frac{2\lambda_1\lambda_2}{\Delta}\left\{R_4 + \frac{i\gamma_2^2}{\lambda_2} R_5\right\}$$
$$R_1 = 5u^2 + (\xi^2 - \eta^2)\cos 2\beta + 2\xi\eta \sin 2\beta, \quad R_2 = \xi\cos\beta + \eta\sin\beta$$
$$R_3 = 2\lambda_1^2 + u^2 + (\xi^2 - \eta^2)\cos 2\beta + 2\xi\eta\sin 2\beta$$
$$R_4 = [2\xi\eta\cos 2\beta - (\xi^2 - \eta^2)\sin 2\beta]\sin\alpha$$
$$R_5 = (\eta\cos\beta - \xi\sin\beta)\cos\beta, \quad \Delta = \gamma_2^4 - 4\lambda_1\lambda_2 u^2$$

The branches of λ_j, when $|u| < \theta_j$ in the expressions given above are chosen as follows /3/:

$$\lambda_j = \sqrt{u^2 - \theta_j^2} = -i\sqrt{\theta_j^2 - u^2}$$

When a more accurate solution of the system of integral equations is needed, the process of constructing the consecutive approximations can be continued.

It should be noted that when the cavity is cylindrical, the system of integral equations has the same structure and properties and its solution is constructed in exactly the same manner.

2. In order to determine the stress-strain state of the elastic medium by solving the system of integral equations, we have the following expressions for the displacement vector components /2/:

$$u_x = u_x^{(1)} + u_x^{(2)}, \quad u_y = u_y^{(1)} + u_y^{(2)}, \quad u_z = u_z^{(1)} + u_z^{(2)} \tag{2.1}$$
$$u_x^{(2)} = u_z^{(2)} \sin\alpha\cos\beta + u_\alpha^{(2)}\cos\alpha\cos\beta - u_\beta^{(2)}\sin\beta, \quad \beta = \operatorname{arctg}\frac{y}{x}$$
$$u_y^{(2)} = u_z^{(2)}\sin\alpha\sin\beta + u_\alpha^{(2)}\cos\alpha\sin\beta + u_\beta^{(2)}\cos\beta,$$
$$\alpha = \operatorname{arctg}\frac{\sqrt{x^2+y^2}}{z+\varepsilon^{-1}}$$
$$u_z^{(2)} = u_r^{(2)}\cos\alpha - u_\alpha^{(2)}\sin\alpha, \quad r = [x^2 + y^2 + (z+\varepsilon^{-1})^2]^{1/2}$$

Here the superscript (1) corresponds to the solution of the problem for a homogeneous elastic half-space, and the superscript (2) corresponds to the three-dimensional problem for an elastic space with a spherical cavity.

When computing the components of the displacement vector (2.1) $u_x^{(1)}, u_y^{(1)}, u_z^{(1)}$, it is sometimes convenient to describe them in terms of the spherical coordinate system R, φ, ψ attached to the centre of the region to which the load is applied. In the adjacent zone we use the numerical algorithm for computing the integrals which determine the components of the displacement vector in an elastic half-space, and in the zone lying away from the region to which the load is applied, it is more convenient to carry out the analysis using asymptotic methods. For example, when $R \gg 1$, we obtain for $0 < \sin^2\varphi < \theta_1^2/\theta_2^2$, $\varphi \neq \pi/2$

$$z \neq 0, \quad R = [(x_0 - x)^2 + (y_0 - y)^2 + z^2]^{1/2}, \quad \varphi = \operatorname{arctg}\frac{z}{\sqrt{R^2 - z^2}} \tag{2.2}$$
$$u_R^{(1)}(R,\varphi,\psi) = -\frac{ap\theta_1^3(2\theta_1^2\sin^2\varphi - \theta_2^2)}{2\pi\mu R\Delta(\xi_1,\eta_1)}\cos\varphi\Phi(\varphi,\psi,\theta_1)\exp(i\theta_1 R) + O(R^{-2})$$
$$u_\varphi^{(1)}(R,\varphi,\psi) = \frac{ap\theta_2^4}{\pi\mu R}\left[\frac{\theta_1^2}{\theta_2^2} - \sin^2\varphi\right]^{1/2}\frac{\cos\varphi\sin\varphi}{\Delta(\xi_2,\eta_2)}\Phi(\varphi,\psi,\theta_2)\exp(i\theta_2 R) + O(R^{-2})$$
$$u_\psi^{(1)}(R,\varphi,\psi) = O(R^{-2}), \quad x_0 = M\varepsilon^{-1}, \quad y_0 = N\varepsilon^{-1}$$
$$\psi = \operatorname{arctg}\frac{x - x_0}{y - y_0}$$

When $\theta_1^2/\theta_2^2 < \sin\varphi < 1$, the form of the relations determining $u_R^{(1)}, u_\psi^{(1)}$ remains unchanged. The expression for $u_\varphi^{(1)}$ becomes

$$u_\varphi^{(1)}(R,\varphi,\psi) = \Phi(\varphi,\psi,\theta_2)\frac{iap\theta_2\cos\varphi\sin\varphi}{\mu\pi R\Delta(\xi_2,\eta_2)}\left[\sin^2\varphi - \frac{\theta_1^2}{\theta_2^2}\right]^{1/2}\exp(i\theta_2 R) + O(R^{-2}) \tag{2.3}$$
$$\Delta(\xi_k,\eta_k) = (2\theta_k^2\sin^2\varphi - \theta_2^2)^2 + 4\theta_k^2\sin^2\varphi[(\theta_1^2 - \theta_k^2\sin^2\varphi)(\theta_2^2 - \theta_k^2\sin^2\varphi)]^{1/2}$$
$$\{\xi_k,\eta_k\} = \{-\theta_k\cos\psi\sin\varphi, -\theta_k\sin\varphi\sin\psi\}$$
$$\Phi(\varphi,\psi,\theta_l) = \frac{4\sin(B\theta_l\sin\varphi\cos\psi)\sin(C\theta_l\sin\varphi\sin\psi)}{\theta_l^2\sin^2\varphi\cos\psi\sin\psi}, \quad l = 1, 2$$

The expressions for $u_R^{(1)}(R,\varphi,\psi), u_\varphi^{(1)}(R,\varphi,\psi), u_\psi^{(1)}(R,\varphi,\psi)$ when $\varphi = \pi/2$ are described in practice by the relations determining the Rayleigh wave (corresponding to the residue at the Rayleigh pole).

Let us write $u_r^{(2)} = u_1^{(2)}$, $u_\alpha^{(2)} = u_2^{(2)}$, $u_\beta^{(2)} = u_3^{(2)}$. In this case we obtain, for $\sqrt{M^2+N^2} \ll 1$,

$$u_k^{(2)}(r,\alpha) = a\sum_{n=0}^{\infty}\frac{1}{\Delta_n}[a_{22}^{(n)}u_{n1}^{(k)}(r,\alpha) - a_{21}^{(n)}u_{n2}^{(k)}(r,\alpha)]Y_{1n} \tag{2.4}$$
$$Y_{1n} = \frac{(-1)^n(2n+1)iep}{\pi\mu}\theta_1 j_n(\theta_1)\exp\left[i\left(\theta_1\varepsilon^{-1} + \frac{n\pi}{2}\right)\right]$$
$$u_{n2}^{(1)}(r,\alpha) = \frac{1}{\theta_2^2 r^{3/2}}H_{n+1/2}^{(1)}(\theta_2 r)P_n(\cos\alpha)$$
$$u_{n1}^{(1)}(r,\alpha) = \frac{1}{\theta_1^2\sqrt{r}}\left[\frac{n}{r}H_{n+1/2}^{(1)}(\theta_1 r) - \theta_1 H_{n+3/2}^{(1)}(\theta_1 r)\right]P_n(\cos\alpha)$$
$$u_{n1}^{(2)}(r,\alpha) = \frac{1}{\theta_1^2 r^{3/2}}H_{n+1/2}^{(1)}(\theta_1 r)\frac{dP_n(\cos\alpha)}{d\alpha}$$
$$u_{n2}^{(2)}(r,\alpha) = \frac{1}{\theta_2^2 n(n+1)\sqrt{r}}\left[\frac{n+1}{r}H_{n+1/2}^{(1)}(\theta_2 r) - \theta_2 H_{n+3/2}^{(1)}(\theta_2 r)\right]\times$$
$$dP_n(\cos\alpha)/d\alpha, \quad u_{n1}^{(3)} = u_{n2}^{(3)} = 0$$

where $j_n(\theta_1) = \sqrt{\pi/(2\theta_1)}\,J_{n+1/2}(\theta_1)$ are spherical Bessel functions of the first kind.

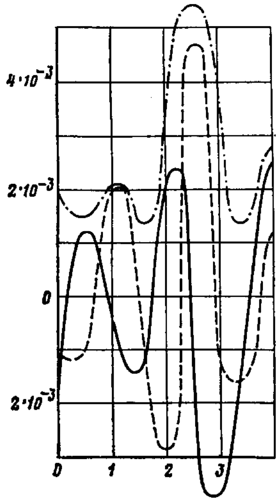

Fig.1

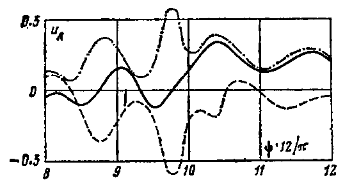

Fig.2

It should be noted that the relations describing the wave field in an elastic medium were obtained on the assumption that $R \gg 1,\ \theta_1/\varepsilon \gg 1,\ \sqrt{M^2+N^2} \ll 1$. When $\varepsilon \ll 1$, the expressions can be used in practice in the region containing a spherical cavity.

The accuracy of the description of the displacement field in an elastic medium is determined by the accuracy of the solution of the system of integral equations (1.2). When analysing the displacement field in the zone near to Ω, the integrals determining $u_R^{(1)}$, $u_\varphi^{(1)}$, $u_\psi^{(1)}$ are calculated directly using a digital computer. The relations determining $u_r^{(2)}$, $u_\alpha^{(2)}$, $u_\beta^{(2)}$ are obtained only under the assumption that $\theta_1/\varepsilon \gg 1,\ \sqrt{M^2+N^2} \ll 1$, and have the form (2.4).

The proposed investigative scheme was realized on a digital computer. A numerical analysis of the displacement field was carried out for practically the whole elastic region. The behaviour of the solutions was studied when various parameters of the problem were varied. In particular, we studied the dependence of the amplitude functions of the displacement of the points of the elastic region on the frequency of the oscillations near to and far from the cavity. The behaviour of the amplitude displacement functions of the angular and radial coordinates was studied at fixed oscillation frequencies.

Figure 1 shows, as an example, the amplitude-frequency characteristics of the displacement u_R of a point of the region whose coordinates are $R = 8.0,\ \psi = 3.927,\ \varphi = 2.186$ ($\nu = 1/_3,\ \varepsilon = 0.1,\ b = c = 10$). The solid line shows the real component and the dashed line the imaginary component, and the dot-dash line the modulus of the amplitude function.

Figure 2 shows the dependence of u_R on the angular coordinate ψ in the plane $\varphi = 2.186$ perpendicular to the boundary of the half-space and passing through the centres of the cavity and the region Ω when $R = 18.4,\ b = c = 10,\ \varepsilon = 0.1,\ \nu = 1/_3,\ \theta_2 = 1$.

Analysis of the solutions obtained shows that when the wave field is determined in the zone far from the cavity $(r \gg 1,\ R \sim 1)$ with an accuracy up to terms $O(\varepsilon)$, we can use the solution of the problem for a homogeneous elastic half-space. In the zone immediately adjacent to the cavity, the solution is determined (with the same degree of accuracy) by the relations (2.1)-(2.4) and can be obtained equally well by solving both auxiliary problems formulated in Sect.1.

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